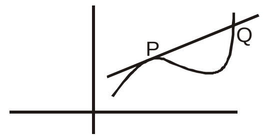
EXERCISE – V HINTS & SOLUTIONS

Sol.1 $\sqrt{2}\,x + y - 2\sqrt{2} = 0$ **or** $\sqrt{2}\,x - y - 2\sqrt{2} = 0$

$y = 2t^3$; $x = 3t^2$

$\frac{dy}{dt} = 6t^2$, $\frac{dx}{dt} = 6t$



$P(3t_1^2, 2t_1^3)$
$Q(3t_2^2, 2t_2^3)$

$\frac{dy}{dx} = \frac{6t^2}{6t} = t$

$\left.\frac{dy}{dx}\right|_P = t_1$

$\left.\frac{dy}{dx}\right|_Q = t_2$

$t_1 t_2 = -1$ ......(1)

slope of PQ $= \frac{2t_2^3 - 2t_1^3}{3t_2^2 - 3t_1^2} = t_1$

$2t_2^2 + 2t_1t_2 + 2t_1^2 = 3t_1(t_2 + t_1)$

$\Rightarrow$ $(t_1 + 2t_2)(t_1 - t_2) = 0$

$t_1 \neq t_2$; $t_1 = -2t_2$ ......(2)

from (1) & (2)

$t_1^2 = 2 \Rightarrow t_1 = \pm\sqrt{2}$

Tangent at P

$y - 2t_1^3 = t_1(x - 3t_1^2)$

put t_1 values

$\sqrt{2}\,x + y - 2\sqrt{2} = 0$ or $\sqrt{2}\,x - y - 2\sqrt{2} = 0$

Sol.2 **1**

Slope of Tangent = $f'(x)$ P(3, 4)

$f'(x)|_p = -\tan\frac{3\pi}{4}$

$f'(3) = 1$

Sol.3 $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$

$y^3 + 3x^2 = 12y$ ....(1)

$3y^2\frac{dy}{dx} + 6x = 12\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{6x}{12 - 3y^2}$

$\frac{dx}{dy} = \frac{12 - 3y^2}{6x} = 0$

$\Rightarrow y = \pm 2$

from (1)

if $y = 2 \Rightarrow$ $x = \pm\frac{y}{\sqrt{3}}$

and if $y = -2 \Rightarrow 3x^2 = -16$ No real solution

Reqd. point $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$

Sol.4 **(–6, –7)**

$y = x^2 + 6$

Tangent at (1, 7)

$\frac{(y+7)}{2} = x + 6$

$y = 2x + 5$ ....(1)

which is also tangent to the circle

$x^2 + y^2 + 16x + 12y + c = 0$

i.e. $x^2 + (2x + 5)^2 + 16x + 12(2x + 5) + c = 0$

must have equal roots i.e. $\alpha = \beta$

$\Rightarrow 5x^2 + 60x + 85 + c = 0$

$\alpha + \beta = -\frac{60}{5}$

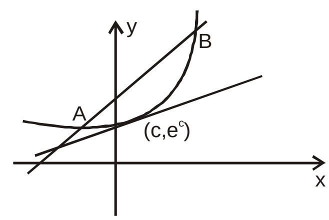
$\alpha = -6$

$\Rightarrow x = -6$ and $y = 2x + 5 = -7$

Q (–6, –7)

Sol.5 slope of the line joining the points $(c-1, e^{c-1})$ and $(c+1, e^{c+1})$ is

$\frac{e^{c+1} - e^{c-1}}{2} > e^c$



$\Rightarrow$ tangent to the curve $y = e^x$ will intersect the given line to the left of the line x = c